19 / 07 / 2023

fojk jk jdr mÍCIKh - 2023
Second Term Examination - 2023

10 | E | I

12 fY%aKsh
GRADE 12

ixhqla; .Ks;h
COMBINED MATHEMATICS

03 meh
03 Hours

- **Answer 06 questions only.**

01. a). Solve the equation $4^{x+1} = 2 - 7\,(2^x)$

b). Solve the equation $2\,x + 3 - \sqrt{x+8} = 2$

c). Prove that,

$$\frac{1}{log_x\,(yz)+1} + \frac{1}{log_y\,(zx)+1} + \frac{1}{log_z\,(xy)+1} = 1$$

d). Resolve into partial fractions, $\frac{2}{x^2\,(x+3)}$

02. a). If $a > 0$ and $b^2 - 4ac < 0$ then show that for all $x \in R$, $ax^2 + bx\ c > 0$.

Find the range of value of $\boldsymbol{m}$, for which
$(m-2)\,x^2 - 3\,(m+2)x + 6m > 0$

b). If $x = p + \sqrt{p^2 + 1}$ then show that,

$x - x^{-1} = 2p$

If the roots of the equation,

$x^2 - 2px - 1 = 0$ are $\propto$ and $\boldsymbol{\beta}$

then find the value of $(\propto^2 - \beta^2)$

c). State the factor theorem of a polynomial $p(x)$.
Given that $(x-1)$ and $(x+2)$ are the factors of the polynomial,
$P(x) = x^4 + ax^3 + bx^2 - 2x - 4$.

i. Show that $a = 3$ and $b = 2$.

ii. Factorize $P(x)$ completly.

03. a). If $Sin\ A = \frac{1}{3}$ and is an abtuse angle, then find the value of the $(\cos A + \sqrt{2}\ \cot A)$

b). Prove that , $\frac{Sin\ \theta}{1+cos\ \theta} + \frac{1+cos\ \theta}{Sin\ \theta} = \frac{2\ sec\ \theta}{tan\ \theta}$.

c). If $A + B + C = \pi$ then prove that,

$$Sin\frac{A}{2} + Sin\frac{B}{2} + Sin\frac{c}{2} = 1 + 4\ sin\left(\frac{\pi - A}{4}\right).\ Sin\left(\frac{\pi - B}{4}\right).\ Sin\left(\frac{\pi - c}{4}\right)$$

d). Find the general solution of the equation,

$$Sin\ 2\theta - 2\ Sin\ \theta - Cos\ \theta + 1 = 0$$

04. a). Evaluate,

i. $\lim_{x \to 1} \frac{1 - x^{-1/3}}{1 - x^{-2/3}}$

ii. $\lim_{x \to o} \frac{1 - cos\ 2x}{x^2}$

b). Find the derivative of the function from the first principle,

$f(x) = \sqrt{2x + 1}$

c). Differentiate with respect to $\boldsymbol{x}$,

i. $y = \sqrt{\frac{x - 4}{x + 4}}$

ii. $y = \frac{1 + Sin\ x}{1 + \cos x}$

iii. $y = e^{\tan x}\ .\ell n\ (x^2 + 2) + Sin^{-1}\ 2x$

05. a). Find the magnitude of two forces such that, when they act at right angle their resultant is of magnitude $\sqrt{13}\ \boldsymbol{N}$ and when the forces act an angle $60°$, their resultant is $\sqrt{19}\ \boldsymbol{N}$.

b). State the lamis theorem of three coplanar forces acting at a point.

A weight W is suspended by two perpendicular light inextensible strings of length $\boldsymbol{a}$ and $\boldsymbol{b}$ from two points at the same horizontal level show that the tension of one string is $\frac{Wa}{\sqrt{a^2 + b^2}}$. Find also the tension of the other string.

06. a). Let $-2\underline{p} + 5\underline{q}$, $7\underline{p} - \underline{q}$ and $\underline{p} + 3\underline{q}$ be the position vectors of three points $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ respectively, with respect to the fixed origin $\boldsymbol{O}$. Where $\underline{\boldsymbol{p}}$ and $\underline{\boldsymbol{q}}$ are non-paralled vectors. Show that $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are collinear. Find the ratio in which $\boldsymbol{C}$ divides $\boldsymbol{AB}$.

b). $\boldsymbol{ABCD}$ is a trapezium of sides $AD = DC = CB = a$ and $AB = 2a$, also $\boldsymbol{AB}$ and $\boldsymbol{DC}$ are parallel. Forces of magnitude $1, 3, 5, 6, 2\sqrt{3}$ Newton act along $\overrightarrow{\boldsymbol{AD}}$, $\overrightarrow{\boldsymbol{DC}}$, $\overrightarrow{\boldsymbol{CB}}$, $\overrightarrow{\boldsymbol{BA}}$ and $\overrightarrow{\boldsymbol{AC}}$ respectively. Find the magnitude and the direction of the resultant.

07. a). A particle is moving in a straight line with a uniform acceleration describes distances $\boldsymbol{p}$ and $\boldsymbol{q}$ in successive interval of time t. Show that the acceleration of the particle is $\frac{q-p}{t^2}$.

b). A particle $\boldsymbol{p}$ is dropped from the top of a cliff of height $\boldsymbol{h}$ meters. After it travelles $\boldsymbol{m}$ meters, then another particle $\boldsymbol{Q}$ is dropped from the point which is $\boldsymbol{n}$ meters below from the top of the cliff. If both particles $\boldsymbol{A}$ and $\boldsymbol{B}$ meet at the foot of the cliff at the same instant, then show that the height of the cliff is, $\frac{(m + n)^2}{4m}$.